Flow of non-Newtonian fluids in open channels

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Note: Many academics and research students were involved in this project; a full list of contributors will appear in the full technical paper.

Bechtel Corporation is a very large construction and engineering company with significant projects in Australia. In a whole range of applications, one important issue for the Mining & Metals branch of Bechtel is the efficient handling and transportation of slurries. Of particular interest to the present project, slurries are often transported over long distances in open channels that operate on a very slight incline, with the flow driven by gravity. A challenge for Bechtel is to be able to predict the flow rates in an open channel environment for a given slurry and channel design. The MISG project concerns these issues.

Which constitutive model is appropriate for slurries?

The striking property of slurries is that, provided they are not overly dilute, they exhibit distinct non-Newtonian behaviour. In other words, the viscosity of a slurry is not a material constant, but instead depends on the shear-rate. To characterise the precise nature of this dependence, Bechtel routinely performs experiments with rheometers that measure the relationship between the shear stress applied to a specimen and the shear rate. This data is then fitted against popular non-Newtonian models, such as the Bingham plastic model, a power-law fluid, or a Herschel-Bulkley model. These models are then used to predict flow behaviour in channels and elsewhere.

The Bingham plastic model just mentioned is characterised by a yield stress. For shear stresses below the yield stress, a Bingham plastic fluid will not flow. In the context of open channel flow, this means that there is a region of sheared flow adjacent to the channel wall and then a plug flow region that occupies the remaining part of the cross-section of the channel. Such plug flow regions are observed in the field, which tends to support the use of the Bingham model. Furthermore, since the Bingham model gives rise to a well-defined interface between the sheared region and the plug flow, it has the advantage of providing mathematical solutions that are easy to interpret. On the other hand, power-law fluids are characterised by an index. When this index is small, then a power-law fluid will flow down an open channel with an almost-plug region. In this case there will be no well-defined interface, and so notions like a plug flow are more difficult to describe objectively. The Herschel-Bulkley model is a generalisation of both the Bingham plastic model and the power-law model.

The project team was given some data for a particular slurry in the form of shear stress versus strain rate measurements over a range of mixture concentrations. We found that each set of data fitted a simple power-law extremely well, with smaller values of the index for larger concentrations. For high concentrations, the index was very low, meaning that the dense slurry was a highly shear-thinning power-law fluid. The close fit to a power-law was a little surprising, as we had expected the slurries to behave like a Bingham plastic model. Indeed, the Industry Representative indicated that this slurry had been characterised as a Bingham fluid by fitting a straight line through the data and measuring the slope and intercept in the usual way.

Exact mathematical solutions for laminar flow

The question of whether the flow is laminar or turbulent is addressed briefly at the end of this report. For laminar flow, a semi-circular cross-sectional channel allows an exact mathematical solution for both the Bingham and power-law models. The key assumption for this geometry is that the slurry is occupying the full channel, so the flow field depends on the radial distance only. For the Bingham case, the exact solution provides a simple formula for the radius of the plug region, which turns out to be instructive, even for other geometries. The key feature of this formula is that it predicts the plug radius is proportional to the yield stress but independent of the radius of the semi-circular channel. For the power-law case, the exact solution provides a velocity profile that exhibits near plug flow, especially for small values of the index.

By revisiting the manner in which the data was fitted to a Bingham fluid, we found that one must use this approach with care. The average shear on the wall of the channel can be computed without reference to the model, and this value gives an indication of what experimental data points should be ignored when fitting the measurements to a straight line. Thus, for a particular cross-sectional radius, a careful fitting gives parameter values for the Bingham model that lead to exact solutions that match closely to the power-law fluid. The conclusion is that, while this particular slurry was clearly a power-law fluid, if an engineer wishes to use a Bingham model, then they must be mindful of the appropriate stresses involved before attempting to fit the data to a straight line.

Numerical solutions for laminar flow

The exact solutions mentioned above apply for the special case of a full open channel of semi-circular cross-section. For other cross-sectional shapes, we must resort to numerical solutions of the governing equations. It turns out that this computational task is more difficult than what one may first guess, for both Bingham and power-law models.

For the Bingham model, the mathematical formulation for the problem of steady flow down a open channel is not well defined. The main problem is that the Bingham model does not provide information about the stress field within the plug region. In order to make computational progress, the Bingham model needs adjusting so that stresses are related to shear rates throughout the flow. One common approach is to use the so-called bi-viscous model, which follows the Bingham model except for small shear rates, at which point it changes to Newtonian. The actual value of the shear rate at which the transition is assumed to occur is arbitrary, and not based on the properties of the actual slurry. This defect of the Bingham model is worth recording.

The project team produced numerical solutions for a slurry flowing steadily down an open channel of rectangular cross-section. The results were very interesting, as they highlighted the role the of plug radius found from the exact solution described above. The solution for the 2×1

rectangular channel had a near-circular plug whose radius was essentially the same as that for the circular cross-section. The solution for a very long and thin channel had a almost-one-dimensional plug whose depth was one half the radius of the circular plug. These calculations suggest a simple model for predicting the plug size and shape for laminar flow in a rectangular cross-section. With this plug size, other key quantities like relative flow rates and hydraulic radii are easily calculated.

Turning now to the numerical solutions with the power-law model, the project team discovered that computing solutions for fluids with a low power-law index was rather challenging. In terms of the model, the difficulties arise from the extremely high apparent viscosities at low shear rates. Regardless, solutions were computed for trapezoidal shaped channels and compared to models for Newtonian fluids. This work needs further investigation.

Further approximations for laminar flow

The project team spent time constructing various approximate solutions with a view to providing meaningful insight into the main problem without resorting to complicated numerical procedures. One of these concerned an open channel whose cross-section was almost circular. This approach leads to a linear problem for the Bingham model, which can be solved exactly. Another concerned the limit of extreme shear-thinning, which provides approximations for the power-law model.

An additional approach that the project team worked on was to treat the slurry as Newtonian and then stipulate a plug region that begins where the shear stresses reach the yield stress. The advantage of this hybrid model is that the flow problems for Newtonian fluids are linear and so can be solved exactly by hand using traditional approaches. This work needs further attention to establish the accuracy of the approximation and to explore how the model can provide insight into the key issues with which Bechtel are concerned.

Turbulent flow regime

The exact and approximate solutions above are based on the assumption that the flow is laminar and unidirectional, that is, the slurry is flowing straight down the channel with no chaotic mixing behaviour. However, if the slurry is moving sufficiently quickly, all or part of the fluid flow may be turbulent. The project team identified that the assumption of laminar flow can produce speeds that are unrealistically high, because the slurries operate in a low viscous regime once pushed beyond the yield stress. Thus turbulent flows will frequently arise in practice.

An aspect of the problem introduced by the Industry Representative was to predict the parameter regime in which turbulence occurs, and to derive formulas for the average velocity when the flow is turbulent. For Newtonian fluids these questions are framed in terms of the dimensionless Reynolds number and friction factor. There is no universally accepted formula for these numbers for non-Newtonian fluids, although there are multiple candidates in the literature.

Although the project team did not advance to the point of deriving quantitative results during the MISG meeting itself, discussion within the team did identify avenues of further work, particularly on the Bingham model. It was reasoned that, even at sufficiently high speeds to induce turbulence, a Bingham plastic would still have a large unsheared plug, with turbulent flow restricted to a small gap between the stationary wall and the moving plug. The Reynolds number for such a flow should therefore be based on the width and characteristic viscosity inside the thin turbulent region. To construct a formula for the average velocity of the flow, the velocity profile within the thin region could be approximated using the law of the wall approximation that is used for Newtonian fluid flow. Further work is needed to derive mathematical results from these preliminary observations.